

Assignment 1 Solution :-

- 1) Based on the size, weight and Mode of operation one can classify the Unmanned Aircraft.
- 2) Endurance of the Unmanned Aircraft depends upon -
 - i) Payload
 - ii) Aerodynamics of UAV
 - iii) Atmospheric conditions
- 3) From the hydrostatic equation the variation of pressure, temperature and density with altitude are derived without considering the variation of acceleration due to gravity with altitude.

4) Given altitude $h = 11 \text{ km}$
 $P_{11} = ?$

Step 1) First we have to find the density at 11 km -
| $\rho = -6.5 \text{ K/km}$

Using

$$\begin{aligned}\frac{P_{11}}{P_{sea}} &= \left(\frac{T_{11}}{T_{sea}} \right)^{-\left[\frac{g}{R_d} + 1 \right]} \\ &= \left(\frac{T_{sea} + \rho \times h_{11}}{T_{sea}} \right)^{-\left[\frac{g}{R_d} + 1 \right]} \\ &= \left(\frac{288.16 - 6.5 \times 11}{288.16} \right)^{-\left[\frac{9.81 \times 1000}{6.5 \times 287} + 1 \right]} \\ &= 0.75187\end{aligned}$$

$$\frac{P_{11}}{P_{sea}} = 0.29685$$

$$P_{11} = 0.29685 \times 1.2256$$

$$\boxed{P_{11} = 0.36362 \text{ kg/m}^3}$$

for 17 km

$$\frac{f_{17}}{f_{11}} = e^{-\frac{g}{RT_{11}} [h_{17} - h_{11}]}$$

$$= e^{-\frac{9.81}{287 \times 216.66} [17 - 11] \times 10^3}$$

$$\frac{f_{17}}{0.36382} = 0.88863854$$

$$\boxed{f_{17} = 0.141394 \text{ kg/m}^3}$$

⑤

Given - UAV flying at altitude = 15 km

total pressure measured by pitot tube $\Rightarrow P_0 = 0.1376772 \text{ N/m}^2$

true speed (V_T) = ?

first we have to find the static pressure at 15 km followed by 11 km.

$$\frac{P_{11}}{P_{\text{sea}}} = \left(\frac{T_{11}}{T_{\text{sea}}} \right)^{-\left[\frac{\gamma}{\gamma-1} \right]}$$

$$= \left(\frac{216.66}{288.16} \right)^{5.25864}$$

$$\frac{P_{11}}{P_{\text{sea}}} = 0.223158$$

$$P_{11} = 1.01325 \times 10^5 \times 0.223158$$

$$\boxed{P_{11} = 0.2261556 \times 10^5 \text{ N/m}^2}$$

$$\frac{P_{15}}{P_{11}} = e^{-\frac{g}{RT_{11}} [h_{15} - h_{11}]}$$

$$= e^{-0.63105}$$

$$\frac{\rho_{15}}{\rho_{11}} = 0.532029 \quad \therefore \rho_{15} = 0.12032 \times 10^5 \text{ N/m}^2$$

$$\rightarrow \text{density at 15 km} \Rightarrow \frac{\rho_{11}}{\rho_{\text{sea}}} = \left(\frac{T_1}{T_{374}} \right)^{-\left[\frac{g}{\alpha R} + 1 \right]}$$

$$\therefore = (0.75187)^{4.25864}$$

$$\therefore \rho_{11} = 0.36382 \text{ kg/m}^3$$

Now using,

$$\frac{\rho_{15}}{\rho_{11}} = \frac{\rho_{15}}{\rho_{11}}$$

$$\therefore \rho_{15} = 0.1935 \text{ kg/m}^3$$

using

$$P_0 = P + \frac{1}{2} \rho v^2$$

$$(P_0)_{15} = P_{15} + \frac{1}{2} \rho_{15} (V_s)^2$$

$$\therefore V_T = \sqrt{\frac{2(P_0 - P)_{15}}{\rho_{15}}}$$

$$\therefore V_T = \sqrt{\frac{2(0.1376772 \times 10^5 - 0.12032 \times 10^5)}{0.1935}}$$

$$\therefore \underline{V_T = 133.9 \text{ m/s}} \rightarrow \text{Nearest answer is } 74.165 \text{ m/s}$$

Ex-6) Temperature at 17 km and 13 km will be same. which is 216.66

$$\therefore \frac{T_{17}}{T_{13}} = \frac{216.66}{216.66} = 1$$

⑦ The density ratio at 20 km and 11 km \Rightarrow ?

We know from Question No. 4

$$\left[\rho_{11} = 0.36382 \text{ kg/m}^3 \right]$$

\Rightarrow density at 20 km can be found using -

$$\frac{\rho_{20}}{\rho_{11}} = e^{-\frac{g}{RT_{11}} [h_{20} - h_{11}]}$$

$$= e^{-\frac{9.81}{287 \times 216.66} [20 - 11] \times 10^3}$$

$$\boxed{\frac{\rho_{20}}{\rho_{11}} = 0.2417}$$

\rightarrow Nearest answer is $\rightarrow 0.259$

~~$\rho_{20} = 0.08826 \text{ kg/m}^3$~~

~~$\frac{\rho_{20}}{\rho_{11}}$~~

⑧ The pressure ratio at 20 km and 10 km \Rightarrow ?

Pressure at 20 km can be found using -

$$\frac{P_{10}}{P_{20}} = \left(\frac{T_{10}}{T_{20}} \right)^{-\left[\frac{g}{R} \right]}$$

$$= 286 \left(\frac{223.14}{288.16} \right)^{5.25864}$$

$$\frac{P_{10}}{P_{20}} = 0.2607$$

$$\boxed{P_{10} = 0.264189 \times 10^5 \text{ N/m}^2}$$

for 20 km

$$\frac{P_{20}}{P_{11}} = \frac{P_{20}}{\rho_{11}} \times \rho_{11} \quad \begin{array}{l} \nearrow \text{from question} \\ \nwarrow \text{from question} \end{array}$$

$$= 0.259292 \times 0.2264556 \times 10^5$$

$$\left[P_{20} = 0.58595 \times 10^5 \text{ N/m}^2 \right]$$

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$$\frac{P_2}{P_1} = \frac{0.058555 \times 10^5}{0.264189 \times 10^5}$$

$$\boxed{\frac{P_2}{P_1} = 0.22179}$$

9) from Question 5 (see the solution of Question 5)

the pressure at 15 km -

$$P_{15} = 0.131899 \times 10^5 \text{ N/m}^2$$

$$\boxed{P_{15} = 0.131839 \times 10^5 \text{ N/m}^2}$$

10) Given - $\frac{h_g - h}{h} = 1\%$

using

$$h = \frac{h_g R}{R + h_g}$$

where \Rightarrow

$h_g \Rightarrow$ geometric altitude

$h \Rightarrow$ geopotential altitude

$R = 6378.14$ (Radius of earth)

$$\Rightarrow h R + h_g h = h_g R$$

$$h_g h = h_g R - h R$$

$$\frac{h_g}{R} = \frac{h_g - h}{h}$$

$$\frac{h_g}{R} = 0.01$$

$$h_g = 0.01 R$$

$$h_g = 0.01 \times 6378.14$$

$$\boxed{h_g = 63.7814 \text{ km}}$$